

Mass-dependent Lorentz Violation and Neutrino Velocity

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Abstract

Motivated by a recent and several earlier measurement results of the neutrino velocity, we attempt to resolve the apparent discrepancies between them from the viewpoint of mass-energy relation in special relativity. It is argued that a complicated tachyonic neutrino model or a mass-dependent Lorentz violation theory can do this job.

PACS numbers:

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I. MOTIVATION

Over the past one hundred years, special relativity has been one cornerstone of modern physics, well-established by innumerable experiments and observations. An outstanding feature of special relativity is a universal upper limit of speed, namely the light speed c in vacuum. Astonishingly, according to a recent result [1], this speed record is broken by neutrinos in the OPERA experiment, confirming an earlier record from the MINOS detector [2].

Before drawing a conclusion, certainly it is most imperative to check further systematic errors in the measurement. But it is also intriguing to tentatively make a concession to experimental data and adjust our theories. This paper is a preliminary attempt to resolve the neutrino velocity anomaly with a complicated tachyonic neutrino model and with a tiny deformation of Einstein's mass-energy relation.

The paper is organized as follows. In Sec. II we summarize main results of the neutrino velocity measurements from literature and remark on some interesting features. In Sec. III, we attempt to explain the velocity data in framework of the standard mass-energy relation by introducing a tachyonic mass to neutrinos. We find the tachyonic neutrino model cannot reconcile all data unless the neutrino mass runs with energy. Sec. IV deviates from the standard mass-energy relation and shows how the wired neutrino velocity can be explained by a scenario of mass-dependent Lorentz violation. Models of this scenario are classified in two categories: democratic models universal for all types of matter, and discriminative models sensitive to particle species. Some open questions are covered in Sec. V.

II. NEUTRINO SPEED RECORDS

According to [1] and references therein, hitherto there are four independent measurements accomplished for neutrino velocity. We summarize their results in Table I. In the table we include neutrino flavors, which is helpful for flavor-dependent explanations of these data.

Needless to say, a superluminal velocity of neutrino is highlighted by the positive results $(v - c)/c > 0$ from recent MINOS and OPERA measurements. It conflicts apparently with basic laws in special relativity. On the other hand, the old Fermilab and SN 1987A results are consistent with special relativity. This is easy to check with Eq. (1) below, provided the neutrino mass is of order 10^{-1} eV/ c^2 . However, since Fermilab and SN 1987A did not tell us the signature of $(v - c)/c > 0$, their results still admit a superluminal velocity of neutrino.

The SN 1987A data is unique. Some comments are in order on this point. First, its source and baseline are different from others. The neutrinos have traveled for about 168 thousand years before detection. Second, in contrast to roughly ten thousands of events in other experiments, only a dozen of events are detected for SN 1987A burst. This leads to a large statistic error not shown in Table I. Third, thanks to the astronomical length scale of "baseline", SN 1987A puts a very stringent limit on neutrino velocity, which is violated by both MINOS and OPERA experiments. Each single trustable event from SN 1987A could put the MINOS and OPERA results in doubt. Reverse the logic, were $(v - c)/c \sim +2 \times 10^{-5}$ true for MeV neutrinos, these neutrinos should have advanced their arrival for years than gamma rays from SN 1987A. The lesson is, when formulating theories, one must pay special attention to the SN 1987A constraint.

The results summarized in Table I exhibit dependence on energy, flavor and time/century/baseline. Such features are illuminating. The purpose of this paper is to

Table I: Summary of neutrino velocity measurements. v is the measured average speed of neutrino and c is the velocity constant of light. The neutrino flavors are mostly identified at sources, but for SN 1987A it is chosen for detectors. Limited by space, we have not given the number of events in each experiment. See references for details.

Experiment	Velocity constraint	Energy range	Flavors	Reference
Fermilab	$ v - c /c < 4 \times 10^{-5}$ (95% CL)	30 to 200 GeV	π/K -decay $\nu, \bar{\nu}$	[3, 4]
SN 1987A	$ v - c /c < 2 \times 10^{-9}$	5 to 40 MeV	$\bar{\nu}_e$ and $\bar{\nu}_\mu, \bar{\nu}_\tau$	[5–7]
MINOS	$(v - c)/c = (5.1 \pm 2.9) \times 10^{-5}$ (68% CL)	~ 3 GeV	ν_μ and $\bar{\nu}_\mu, \nu_e, \bar{\nu}_e$	[2]
OPERA	$(v - c)/c = (2.48 \pm 0.28 \pm 0.30) \times 10^{-5}$ (6.0σ)	~ 17 GeV	ν_μ and $\bar{\nu}_\mu, \nu_e, \bar{\nu}_e$	[1]

explore the energy dependence from the viewpoint of mass-energy relation and Lorentz symmetry. We will adhere to Einstein’s mass-energy relation in Sec. III and deviate from it slightly in Sec. IV.

III. TROUBLES FOR TACHYONIC NEUTRINO MODELS

A superluminal velocity can be easily achieved by tachyon with a negative value of squared mass. At the same time, the neutrino mass has been an enigma for half a century. Neutrino oscillation experiments measure the difference of squared masses between orthogonal mass eigenstates. Therefore, it sounds natural to consider neutrinos as tachyons in light of the superluminal phenomenon¹. In this section, we will first develop the simplest version of this idea and then rule it out. After that, we will pursue the possibility of a tachyonic neutrino mass running with energy.

For convenience, we write down Einstein’s mass-energy relation in the form

$$1 - \frac{v^2}{c^2} = \frac{m^2 c^4}{E^2}, \quad (1)$$

which relates energy E and rest mass m of a particle via its velocity v and the light velocity constant c . For ordinary particles, nonnegativity of the right hand side dictates on the left hand side $v \leq c$ as a well-known feature of special relativity. Inversely, violation of this bound can be realized if we extrapolate relation (1) to tachyons with $m^2 < 0$. For small deviation $|v - c|/c \ll 1$, (1) reduces to $2(v - c)/c \sim -m^2 c^4/E^2$. Applied to data, it gives $-m^2 \sim (120 \text{ MeV}/c^2)^2$ for OPERA, $-m^2 \sim (30 \text{ MeV}/c^2)^2$ for MINOS, $|m^2| < (0.32 \text{ keV}/c^2)^2$ for SN 1987A and $|m^2| < (0.27 \text{ GeV}/c^2)^2$ for Fermilab. In the simplest tachyon model, a constant tachyonic mass of neutrino cannot explain all of the results in Table I. Within this model, the OPERA and MINOS results are marginally at odds with each other, and both of them conflict obviously with the SN 1987A bound.

But one may still imagine that, instead of a constant parameter, m^2 is running with energy E . Because of an unknown mechanism, its running is much larger than the prediction of standard model of particle physics. Then one may reconcile the discrepancies mentioned above by tuning the dependence of m^2 on E . Specifically, one might be able to tune the

¹ The hypothesis of tachyonic neutrinos was previously employed to explain certain anomalies in β -decay experiments [8], but the “mass” scale there is much lower than those in this section.

running so that $dm^2/d\ln E < 0$ for $E < E_c$ and $dm^2/d\ln E > 0$ for $E > E_c$. This possibility overlaps partly the discriminative models of mass-dependent Lorentz violation in Sec. IV B. An illustrative example and more discussion will be given there.

We warn that tachyonic modes, like ghosts, generally give rise to instabilities in quantum field theory. A tachyonic neutrino model suffers from the same problem, no matter m^2 is a constant or not. Therefore, it is challenging to construct viable models of this type.

IV. MASS-DEPENDENT LORENTZ VIOLATION

As we have seen in the previous section, a tachyonic mass of neutrino cannot explain observational data in a simple way. So let us try another scenario by deviating from the Lorentz symmetry slightly. Before doing this, we remind the readers that Lorentz invariance is a symmetry of Minkowski metric and that Lorentz boosts are related to some spacetime-dependent conserved quantities. Theoretically, a deformation of Lorentz symmetry can be realized by “curving” the Minkowski spacetime. Observationally, the violation of several conservation laws sheds light on testing our scenario in other approaches.

The main insight is that results in Table I can be accommodated with a deformation of mass-energy relation (1) as

$$1 - \frac{v^2}{c^2} = \lambda - f(\lambda), \quad (2)$$

where $\lambda = m^2 c^4 / E^2$ and $f(\lambda)$ is an unknown function with the property

$$f(\lambda) = \begin{cases} +\mathcal{O}(10^{-5}), & \text{for } \lambda \text{ around } \lambda_c \sim \frac{m_\nu^2 c^4}{(10 \text{ GeV})^2}, \\ 0, & \text{elsewhere.} \end{cases} \quad (3)$$

The deformed relation has a physical interpretation as follows. For any particle of mass m_i , there is a critical energy scale

$$E_{ci} = \frac{m_i c^2}{\sqrt{\lambda_c}} \quad (4)$$

around which the Lorentz symmetry is broken down. The amount of deviation from Lorentz symmetry is determined by $f(\lambda)$. In viewpoint of the standard mass-energy relation, the particle gains a tachyonic effective mass near its critical energy scale. Compared with other scenarios of Lorentz violation, a new feature here is the dependence on mass-energy ratio $\sqrt{\lambda}$. In other words, the Lorentz violation is mass-dependent in this scenario. More interestingly, for massless photons, relation (2) reduces to the standard mass-energy relation if we promote velocity to momentum via $v/c = p/E$.

Based upon the nature of function $f(\lambda)$, this scenario can be divided into two branches. In the first branch, $f(\lambda)$ is a universal function for all species of matter. Both the form and parameters of this function are universal. Then λ_c is a universal constant. We will use “democratic models” to name models in this branch and study them in Sec. IV A. Models in the second branch will be dubbed “discriminative models”. In these models, the exact form and parameters of function $f(\lambda)$ depend on species of matter. For instance, the function $f(\lambda)$ could involve charges and coupling constants of the particle. The discriminative models are investigated in Sec. IV B.

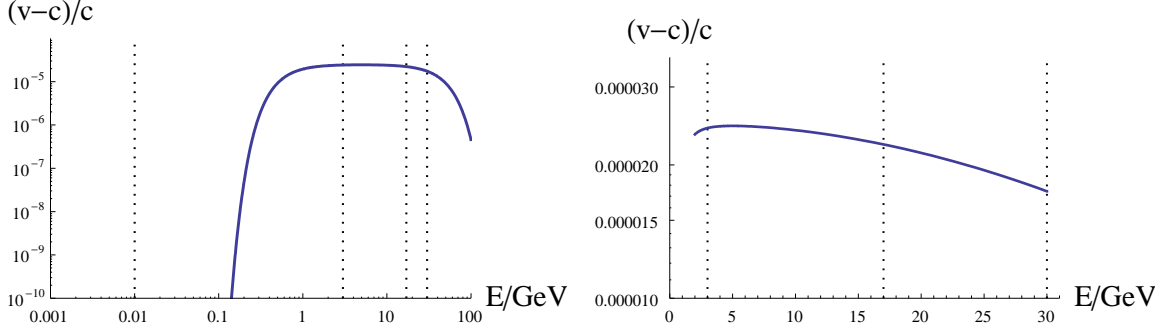


Figure 1: (color online). A numeric illustration of the deformed mass-energy relation (2) with $f(\lambda)$ given by the toy model (5). The model parameters are chosen as $m_\nu = 10^{-1}$ eV/ c^2 , $\epsilon = 0.01$, $\delta = 5 \times 10^{-5}$ and $E_{c\nu} = 5$ GeV. It accommodates well the data listed in Table I.

A. Democratic Models

Most economically $f(\lambda)$ is a universal function for all species of matter irrespective of masses, flavors, charges, *etc.* Then λ_c is approximated by a universal constant. Assuming $m_\nu \sim 10^{-1}$ eV/ c^2 and the critical energy scale $E_{c\nu} \sim 10$ GeV for neutrino, we get $\sqrt{\lambda_c} \sim 10^{-11}$. For electron of mass 0.5 MeV/ c^2 , this yields a critical energy $E_{ce} \sim 10^7$ GeV, a scale well above the CERN LHC energy. For heavier particles, the critical scale is even higher. In this perspective, the mass hierarchy of standard model particles is responsible for the energy hierarchy of Lorentz-violating new physics. And quite fortunately, accelerator-generated neutrinos fall in such a weird “new physics” scale. The hierarchy also puts our scenario in safety. So far we have failed to find a counter example for this scenario. But it is believed that probably the scenario of this branch can be ruled out by a clever reasoning or by some data in the large body of experiments.

Lacking of a counter example temporarily, let us take a closer look at this scenario. To get some sense of it, we devised a toy model of three independent parameters

$$f(\lambda) = \delta \times \exp \left[-\epsilon \left(\frac{\lambda}{\lambda_c} + \frac{\lambda_c}{\lambda} \right) \right] = \delta \times \exp \left[-\epsilon \left(\frac{E_c^2}{E^2} + \frac{E^2}{E_c^2} \right) \right]. \quad (5)$$

Setting $m_\nu = 10^{-1}$ eV/ c^2 , $\epsilon = 0.01$, $\delta = 5 \times 10^{-5}$ and $E_{c\nu} = 5$ GeV, we plot in Fig. 1 the dependence of $(v - c)/c$ on E . It agrees well with the observational constraints in Table I. As illustrated in Fig. 1, parameters $E_{c\nu}$ (or equivalently λ_c) and ϵ dictate the location and width in energy for Lorentz violation. The magnitude of $f(\lambda)$ or $(v - c)/c$ near $E = E_{c\nu}$ is controlled by δ . Here the values of parameters are put and tuned by hand. If there are more data points, we may numerically compute the best-fit values of parameters. As indicated by the smallness of δ and λ_c , we need a fine-tuning of parameters in this toy model. In our scenario, the fine-tuning problem arises from the energy scale and amplitude of Lorentz violation.

The above toy model is helpful phenomenologically but useless in construction of a fundamental theory. Although Lorentz symmetry is broken at the critical scale, we expect a new symmetry will emerge hereabout. The expected new symmetry should be incorporated in the theory by devising a more elegant function $f(\lambda)$. The physical origin of democratic models and new symmetry will be pondered over in Sec. V.

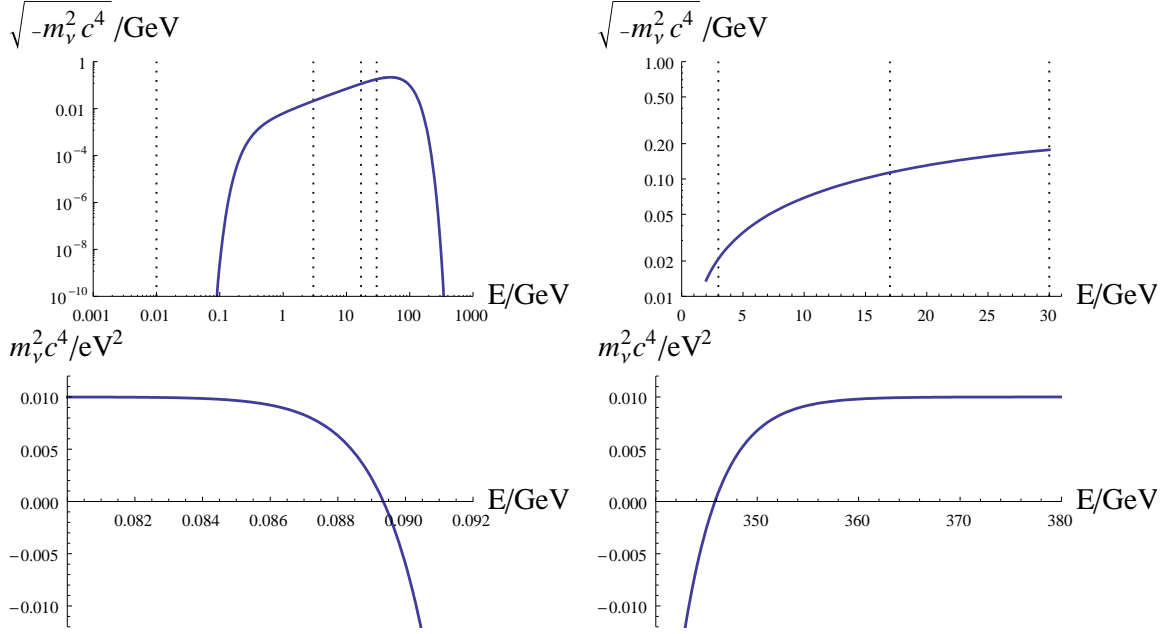


Figure 2: (color online). The running of neutrino mass at relatively high energy according to (6). The model parameters are chosen as $m_\nu(0) = 10^{-1} \text{ eV}/c^2$, $\epsilon_\nu = 0.01$, $\delta_\nu = 5 \times 10^{-5}$ and $E_{c\nu} = 5 \text{ GeV}$. It is in agreement with our expectation in Sec. III.

B. Discriminative Models

A more flexible possibility is that the deformed mass-energy relation is not universal but varies from particle to particle. That is to say, function $f(\lambda)$ involves more parameters such as couplings, charges, *etc.* In particular, it is possible that $f_\nu(\lambda) > 0$ for neutrinos but $f_i(\lambda) \leq 0$ for other species of particles. The subindex implies the species-dependence. We feel that this class of models have more chances to survive than democratic models.

To work with a concrete model, we return to the tachyonic neutrino with a running mass. Other species are irrelevant in the present analysis, because $f_i(\lambda)$ for them can be tuned independently to comply with experiments. Thus we concentrate on the neutrino sector and phenomenologically apply the toy model (5) to it. After some calculation, we find this can be realized if the effective squared mass runs like

$$m_\nu^2(E) = m_\nu^2(0) - \frac{E^2}{c^4} f_\nu(\lambda) = m_\nu^2(0) - \frac{E^2}{c^4} \times \delta_\nu \times \exp \left[-\epsilon_\nu \left(\frac{E_{c\nu}^2}{E^2} + \frac{E^2}{E_{c\nu}^2} \right) \right]. \quad (6)$$

With parameters similar to that in Fig. 1, this model fit well the given measurement results, as is depicted in Fig. 2. One may compute the running of squared mass for this model straightforwardly, which changes signature near $E_{c\nu}$. In model (6), $m_\nu^2(E)$ crosses zero twice. The picture of this process was shot and shown in Fig. 2. It indicates that as energy increases, the neutrino runs from an ordinary particle to a tachyonic particle for a while and then back to an ordinary particle.

V. DISCUSSION

In Sec. IV, we accommodated the neutrino velocity anomaly in the scenario of mass-dependent Lorentz violation. But we did not explain the physical origins of such a behavior of Lorentz violation, which remains an open problem. Here are some thoughts on this problem. As we can expect, there are two possible origins. One is from the nature of spacetime. This happens in democratic models. In this case, we would have to ultimately replace the Lorentz symmetry with some new symmetry, just like switching from Galilean symmetry to Lorentz symmetry. This will deform plane wave solutions and would potentially bring troubles to quantization. It reminds us of doubly special relativity [9], κ -Minkowski spacetime [10] and very special relativity [11]. The other origin is from the dynamics of matter fields, *e.g.* the running of mass with energy. This happens in both democratic models and discriminative models. In this case, the Lorentz symmetry is not broken for spacetime, but it is apparently broken for certain species of matter at special energy scales.

The results in this paper are tentative rather than conclusive. First, our investigation has not exhausted all potential explanations for the superluminal propagation of neutrinos. There could be reasonable explanations making use of flavor dependence or time/century/baseline dependence of the data. They are out of the scope this work although we have highlighted these features in Sec. II. Second, as we emphasized in the very beginning, before drawing a conclusion about fundamental theories, the most imperative thing is to further check the systematic errors in measurement. We are waiting for future news from experiments before polishing our models.

When this work was near completion, related papers [12, 13] appeared, also tackling with the issues of neutrino velocity data in Table I. Previous investigations related to neutrino velocity and Lorentz violation can be found from [14–24] and references therein.

Acknowledgments

We are grateful to Xiao-Dong Li and Shuang Wang for valuable discussion. TW would like to thank the Kavli Institute for Theoretical Physics China for hospitality where this work was done during a program on String Phenomenology and Cosmology. ML is supported by the NSFC grants No.10535060, No.10975172 and No.10821504, and by the 973 program grant No.2007CB815401 of the Ministry of Science and Technology of China. TW is supported by the NSFC grant No.11105053.

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- [1] OPERA, arXiv:1109.4897 [hep-ex].
 - [2] P. Adamson *et al.* [MINOS Collaboration], Phys. Rev. D **76**, 072005 (2007) [arXiv:0706.0437 [hep-ex]].
 - [3] G. R. Kalbfleisch, N. Baggett, E. C. Fowler and J. Alspector, Phys. Rev. Lett. **43**, 1361 (1979).
 - [4] J. Alspector *et al.*, Phys. Rev. Lett. **36**, 837 (1976).
 - [5] M. J. Longo, Phys. Rev. D **36**, 3276 (1987).
 - [6] K. Hirata *et al.* [KAMIOKANDE-II Collaboration], Phys. Rev. Lett. **58**, 1490 (1987).
 - [7] R. M. Bionta *et al.*, Phys. Rev. Lett. **58**, 1494 (1987).
 - [8] J. Ciborowski and J. Rembielinski, arXiv:hep-ph/9607477.

- [9] G. Amelino-Camelia, *Nature* **418**, 34 (2002) [arXiv:gr-qc/0207049].
- [10] A. Agostini, arXiv:hep-th/0312305.
- [11] A. G. Cohen and S. L. Glashow, *Phys. Rev. Lett.* **97**, 021601 (2006) [arXiv:hep-ph/0601236].
- [12] G. Cacciapaglia, A. Deandrea and L. Panizzi, arXiv:1109.4980 [hep-ph].
- [13] G. Amelino-Camelia, G. Gubitosi, N. Loret, F. Mercati, G. Rosati and P. Lipari, arXiv:1109.5172 [hep-ph].
- [14] J. R. Ellis, N. Harries, A. Mereaglia, A. Rubbia, A. Sakharov, *Phys. Rev.* **D78** (2008) 033013. [arXiv:0805.0253 [hep-ph]].
- [15] Y. Grossman, C. Kilic, J. Thaler and D. G. E. Walker, *Phys. Rev. D* **72**, 125001 (2005) [hep-ph/0506216].
- [16] J. Thierry-Mieg,
- [17] V. A. Kostelecky and M. Mewes, *Phys. Rev. D* **69**, 016005 (2004) [arXiv:hep-ph/0309025].
- [18] D. Colladay and V. A. Kostelecky, *Phys. Rev. D* **58**, 116002 (1998) [arXiv:hep-ph/9809521].
- [19] V. A. Kostelecky and N. Russell, *Rev. Mod. Phys.* **83**, 11 (2011) [arXiv:0801.0287 [hep-ph]].
- [20] J. S. Diaz, arXiv:1109.4620 [hep-ph].
- [21] H. Pas, S. Pakvasa and T. J. Weiler, *Phys. Rev. D* **72**, 095017 (2005) [arXiv:hep-ph/0504096].
- [22] J. Dent, H. Pas, S. Pakvasa and T. J. Weiler, arXiv:0710.2524 [hep-ph].
- [23] S. Hollenberg, O. Micu, H. Pas and T. J. Weiler, *Phys. Rev. D* **80**, 093005 (2009) [arXiv:0906.0150 [hep-ph]].
- [24] J. Alfaro, *Phys. Rev. Lett.* **94**, 221302 (2005) [arXiv:hep-th/0412295].